Edexcel Physics A-level

Topic 12: Gravitational Fields

Notes

12 - Gravitational Fields

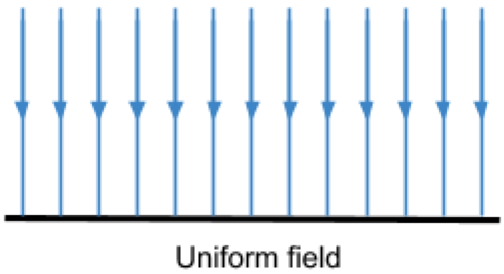
12.174 - Gravitational fields

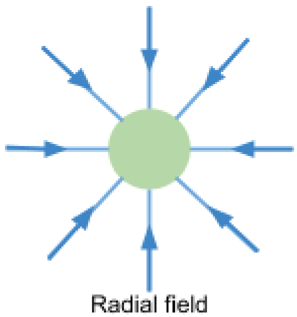
A **force field** is an area in which an object experiences a **non-contact force**. Force fields can be represented as **vectors**, which describe the direction of the force that would be exerted on the object, from this knowledge you can deduce the direction of the field. They can also be represented as diagrams containing **field lines**, the distance between field lines represents the strength of the force exerted by the field in that region.

A gravitational **field** is a **force field** in which objects with mass experience a force.

12.175 - Gravitational field strength

There are two types of gravitational field; a **uniform field** or **radial field**. These can be represented as the following field lines:





The arrows on the field lines show the direction that a force acts on a mass. A **uniform field** exerts the **same** gravitational force on a mass everywhere in the field, as shown by the parallel and equally spaced field lines. In a **radial field** the **force exerted depends on the position** of the object in the field, e.g in the diagram above, as an object moves further away from the centre, the magnitude of force would decrease because the distance between field lines increases. The Earth's gravitational field is radial, however very close to the surface it is almost completely uniform.

Gravitational field strength (g) is the force per unit mass exerted by a gravitational field on an object. This value is constant in a uniform field, but varies in a radial field. The general formula for calculating the gravitational field strength is:

$$g = \frac{F}{m}$$

Where **F** is the force exerted on the object and **m** is the mass of the object.

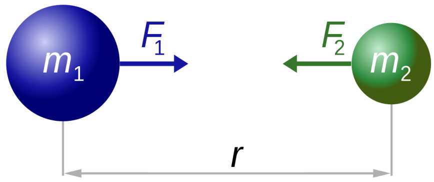
12.176 - Newton's law of universal gravitation

Gravity acts on any objects which have **mass** and is **always attractive**.

Newton's law of gravitation shows that the magnitude of the gravitational force between two masses is **directly proportional to the product of the masses**, and is **inversely proportional to the square of the distance between them**, (where the distance is measured between the two centres of the masses).

$$F = \frac{Gm_1m_2}{r^2}$$

Where **G** is the gravitational constant, $\mathbf{m_1/m_2}$ are masses and **r** is the distance between the centre of the masses.

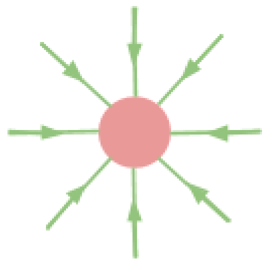


$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

Image source: I, Dennis Nilsson, CC BY 3.0

12.177 - Gravitational field strength in a radial field

An example of a radial gravitational field, is the one formed by a point mass.



The gravitational field strength (g) in a radial field **varies** and you can derive an equation for the gravitational field strength at a point in the field using Newton's law of gravitation and the general formula for **gravitational field strength** as shown below:

$$F = \frac{Gm_1m_2}{r^2} \qquad g = \frac{F}{m_2}$$

Rearrange the general formula so that its subject is the force exerted (F). Note that this just gives us the equation for calculating the weight of an object, where m_2 is its mass.

$$F = m_2g$$

You can now set Newton's law of gravitation and the equation above equal to each other, and cancel down to get an equation for gravitational field strength.

$$m_2g = \frac{Gm_1m_2}{r^2}$$

$$g = \frac{Gm_1}{r^2}$$

Where m_1 is the mass of the object creating the gravitational field.
This can be re-written more clearly as:

$$g = \frac{Gm}{r^2}$$

Where **g** is the gravitational field strength, **G** is the gravitational constant, **m** is the mass of the object causing the gravitational field and **r** is the distance from the centre of the mass.

12.178 - Gravitational potential

Gravitational potential (V) at a point is the work done per unit mass when moving an object from infinity to that point. Gravitational potential **at infinity is zero**, and as an object moves from infinity to a point, energy is released as the gravitational potential energy is reduced, therefore **gravitational potential is always negative**.

$$V = -\frac{GM}{r}$$ (For a radial field)

Where **M** is the mass of the object causing the field, **r** is the distance between the centres of the objects.

The **gravitational potential difference(ΔV)** is the energy needed to move a unit mass between two points and therefore can be used to find the work done when moving an object in a gravitational field.

$$Work\ done = m\Delta V$$ Where **m** is the mass of the object moved.

12.179 - Comparing electric and gravitational fields

The table below describes some of the similarities and differences between electric and gravitational fields:

Similarities	Differences
Forces both follow an inverse-square law	In gravitational fields, the force exerted is always attractive, while in electric fields the force can be either repulsive or attractive.
Use field lines to be represented and can both be either uniform or radial	
Use equations of a similar form to find the force exerted and field strength (though use different values)	Electric force acts on charge, while gravitational force acts on mass.

12.180 - Orbital motion

Kepler's third law is that the **square of the orbital period (T) is directly proportional to the cube of the radius (r)**: $T^2 \propto r^3$. This can be derived through the following process:

1. When an object orbits a mass, it experiences a **gravitational force** towards the centre of the mass, and as the object is moving in a circle, this gravitational force acts as the **centripetal force**. Therefore we can **equate** the equations of centripetal force and gravitational force:

$$Centripetal\ force = \frac{mv^2}{r} \qquad Gravitational\ force = \frac{GMm}{r^2}$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

2. Rearrange the equation to make it v^2 the subject.

$$v^2 = \frac{GM}{r}$$

3. Velocity is the rate of change of displacement, therefore you can find v in terms of radius (r) and orbital period (T):

$$v = \frac{2\pi r}{T} \quad \Rightarrow \quad v^2 = \frac{4\pi^2 r^2}{T^2}$$

Because the diameter of a circle is $\boldsymbol{2\pi r}$, and the object will travel this distance in one orbital period.

4. Substitute the equation for v^2 in terms of r and T, into the original equation (from step 2).

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

5. Rearrange to make T^2 the subject.

$$T^2 = \frac{4\pi^2}{GM} \times r^3$$

As $\frac{4\pi^2}{GM}$ is a constant, this shows that $T^2 \propto r^3$ (**T^2 is directly proportional to r^3**).

You must be able to apply Newton's laws of motions and gravitation to orbital motion as shown above. It is important to keep in mind that if an orbit is circular, all the equations you learnt to do with **circular motion** will apply.